A NEW METHOD FOR THE TRANSIENT SIMULATION OF CAUSAL LINEAR SYSTEMS DESCRIBED IN THE FREQUENCY DOMAIN

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Abstract

A convolution-based method is described for the transient analysis of causal linear systems. The main novelty lies in the method proposed for computing discrete impulse response samples, which possess excellent frequency interpolation properties to the system function, even though comparatively few samples are used. Convolution operations are accordingly highly efficient, and results are presented to validate the method in comparison to (a) theoretical analysis, (b) SPICE simulation, and (c) experimental step-response results for a lossy microstrip filter. Extensions of the method to more general nonlinear transient simulation are conceptually straightforward.

Introduction

There is a continuing interest amoung the microwave CAD community in the development of efficient numerical procedures for the transient analysis of linear systems subject to arbitrary excitation signals (e.g. [1]-[4]). A standard technique in such cases is based on the Discrete Fourier Transform (DFT), which is usually encountered in the form of its efficient algorithmic implementation as the Fast Fourier Transform (FFT). However, due to its inherent periodicity, the DFT frequently has serious limitations in practice, and may produce erroneous results due to aliasing etc. if not used with some care [6].

The purpose of this contribution is to introduce a modified mathematical transformation to interconnect the time and frequency domains, which may still be implemented numerically using the FFT algorithm, and provides a powerful basis for addressing the transient analysis of both purely-linear and mixed nonlinear/linear systems, provided the usual physical condition of system causality is satisfied. The method described here enables a dramatic improvement in the speed and accuracy of operations connecting the time-domain and the frequency-domain, and leads to a versatile, general-purpose method for performing the transient analysis of causal linear systems.

Transient Analysis of Linear and Nonlinear Systems

The transient analysis of linear systems arises in a variety of contexts. In one type of situation, the terminating circuits on the linear systems are themselves linear, and, for example, the problem to be solved could involve a study of the propagation of a digital pulse along lossy, dispersive, and perhaps coupled, transmission line interconnects with resistive terminations.

A second case is the familiar problem in nonlinear microwave simulation in which it is required to analyse an interconnection of two sub-systems, one of which is most usefully described in the time-domain (e.g. arising from device large-signal models), while the other is most naturally described in the frequency-domain (e.g. describing the external circuit environment). Harmonic Balance [5] is a powerful technique in such cases if the excitation signal is periodic, while programs such as SPICE can handle the transient analysis of certain types of idealised distributed circuits, even though fairly inefficiently. However, it is much more difficult to find a useful, general technique when one desires the true transient response of a nonlinear system with a realistic type of distributed external circuit behaviour with loss, dispersion, and so on.

For simplicity, the examples quoted in the following are confined to purely linear systems, but the development of the method presented here was originally motivated by the requirements of mixed nonlinear/linear system simulation. In fact, although not described here, the method has also been extensively tested in nonlinear simulation applications, and extremely satisfactory results have been obtained.

The Proposed Approach to the Transient Analysis of Linear Systems

In order to focus the discussion, we will consider the simple example in Fig. 1, where it is assumed that the current response is required for an arbitrary voltage excitation. A value for the system function S11(f) is assumed to be available for any given frequency 'f', where the system function is just the voltage reflection coefficient (referred to 'Ro'):

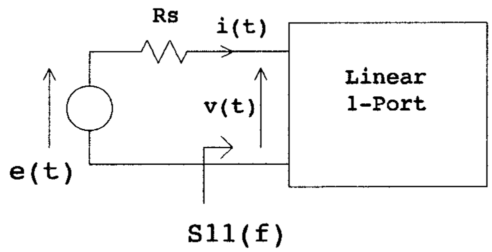


Fig. 1. 1-Port Test Circuit

In the frequency domain:

$$V(f) - I(f).R_o = S_{11}(f).[V(f) + I(f).R_o] \quad ...(1)$$

If one wishes to compute i(t) for a given e(t), then samples of e(t) may be transformed to E(f) using the DFT, followed by a solution of the complex equation (1) together with Kirchhoff's voltage law for the generator loop, yielding values for I(f), and an inverse DFT may be used to produce samples of i(t). This procedure is exact provided e(t) is band-limited and sampled at least at its Nyquist frequency. However, in many practical cases, e(t) is not band-limited (e.g. it is a unit step), and in these cases application of the DFT leads to aliasing errors which are difficult to quantify and require large transform sizes for their minimisation.

Suppose now we take the inverse Fourier Transform of Eq.(1):

$$v(t) - i(t).R_o = h_{11}(t)*(v(t) + i(t).R_o) \quad ...(2)$$

where '*' indicates a convolution. If samples of the impulse response function h11(t) were available, the convolution integral could be evaluated numerically, and approximate sample values could then be obtained for i(t). The difficulty is of course calculating h11(t). Any attempt to bandlimit S11(f) in order to compute h11(t) numerically, produces an impulse response which is non-causal.

The approach adopted here begins with the computation of S11(f) over some band [0, fm], where the source spectral energy is assumed to be relatively small beyond fm. One then forms the periodic extension of the function S11(f) over the entire frequency axis. For the present, let us assume that S11(f) has zero imaginary part at f = fm, so that when S11(f), together with its Hermitean part defined for [-fm, 0], is periodically extended, it forms a smooth, complex-valued function with period [2.fm]. (If the zero imaginary part condition is not met, a uniform delay can be introduced to achieve it, the effect of which may be readily removed later through a simple shift operation in the time-domain).

From signal processing theory, it is known that the impulse response function now becomes truly a discrete (real-valued) function, and the convolution integrals in Eq.(2) are then exactly replacable by summations. In principle, the impulse response values may be determined by a Fourier Series operation on the periodic system function, however this provides impulse response values for both negative time and positive time. The novelty in the method proposed here centres on the observation that for a causal system function, the real and imaginary parts are related by a Hilbert transform, and the impulse response is zero-valued for negative time. These conditions are now forced in the following transform pair, which are proposed as a replacement for the standard Fourier Series approach:

$$h_{11}(nT) = \frac{T}{2\pi} \cdot \int_{-\omega_m/2\pi}^{\omega_m/2\pi} S_{11}(\omega).\exp[+jn\omega T].d\omega$$

$$S_{11}(\omega) = \sum_{n=0}^{+\infty} h_{11}(nT).\exp[-jn\omega T] \quad ...(3)$$

where T = 1 / (2.fm). The detailed justification for this transform pair is not given here for lack of space, but this formulation is the key to the method proposed. In practical implementation, the integral in Eq.(3) must be evaluated numerically from samples of S11(f), however, both forward and reverse transforms may be readily evaluated using an FFT formulation, after appropriate manipulations have been carried out.

The crucial question then becomes related to the interpolation properties of the impulse response samples generated, i.e. for a given finite number of samples 'N', how well does the summation in Eq.(3) describe the original system function between sample points? For example, it is well known from digital filter design, that the interpolation behaviour of the DFT between sample points is often very poor in practice. However, a remarkable property of the transform pair given in Eq.(3) is that the interpolation properties of the impulse response samples are extremely good, and surprisingly small numbers of samples (typically of the order of 100 or less) are sufficient to describe even highly-complex system function behaviour with excellent accuracy over a wide frequency range. This in turn makes the computation of the convolutions such as in Eq.(2) above very economical, and leads to a highly efficient method for the transient simulation of system response for essentially arbitrary excitation waveforms. It should be clear that the extension of this procedure to mixed nonlinear / linear simulation is quite straightforward.

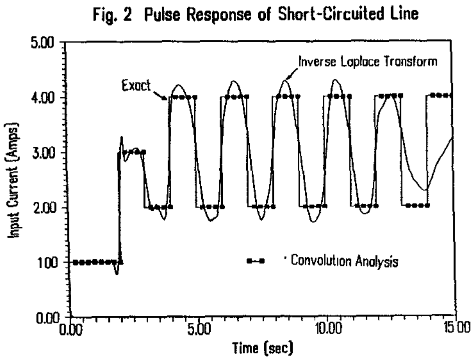
Examples of Applications

Numerous exercises have been carried out to validate and test the capabilities of the above method for the transient analysis of linear systems and only a few examples are presented in the following:

(a) Pulse Response of Short-Circuited Lossless Line:

This example was given recently by Griffith and Nakhla [4], and is effectively the circuit of Fig. 1, with Rs = 0, and the linear 1-port in the simple form of a lossless, short-circuited 1-ohm line with a time delay of 1 sec. The transient input current is sought in response to a unit voltage pulse excitation of 3 sec. duration. Figure 2 shows the exact result, together with results from both the method in [4] based on a numerical inversion of the Laplace Transform, and the convolution method given here. As discussed in [4], this example is not readily solved by DFT techniques, but it is clear that the present method gives results which are in almost perfect agreement with the exact solution.

Fig. 2 Pulse Response of Short-Circuited Line



(b) Step-Response of a Lossy Microstrip Low-Pass Filter

The equivalent circuit of this filter is given in Fig. 3(a), and since the filter was realised in practice on glass-epoxy board, the transmission lines are modelled with significant loss and dispersion. Figure 3(b) shows the impulse response samples computed from S21(f) using the method described here, and Fig. 3(c) indicates the excellent interpolation properties of these samples in the continuous-frequency domain, even though only 64 values are being used.

Fig. 3(b) Impulse Response Samples [S21(f)]

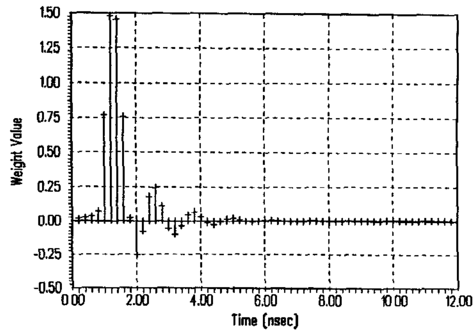


Fig. 3(c) Interpolation Properties of Impulse Response Samples for Lossy Microstrip Filter

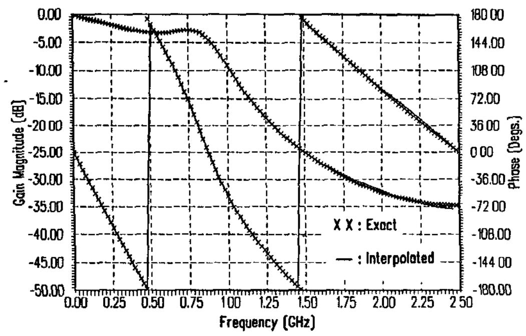


Figure 4 (a) shows the voltage step response at the input computed both using the method given here, and using the SPICE program, but assuming zero transmission line loss and dispersion in each case. An open-circuit load condition is assumed, and the step rise-time is 40 psec. Excellent agreement is obtained, although it should be noted that the computer time required by the present method is more than 8 times less than that required by SPICE. Finally, Fig. 4 (b) shows a comparison between the results of a convolution analysis including loss and dispersion, and the experimental step response determined using a HP 54123T 34-GHz oscilloscope system. The simulated results in this case are seen to be in very good agreement with measured results from the fabricated filter.

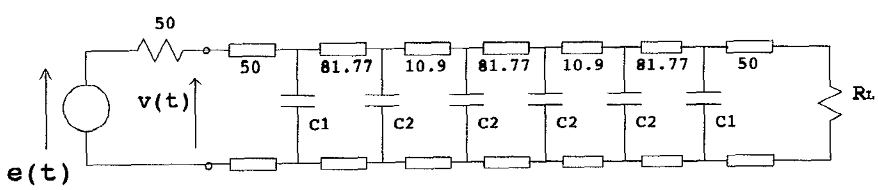


Fig. 3(a) Microstrip Low-Pass Filter

Fig. 4(a) Step Response of Lossless L.P. Filter: SPICE and Convolution Analysis Compared

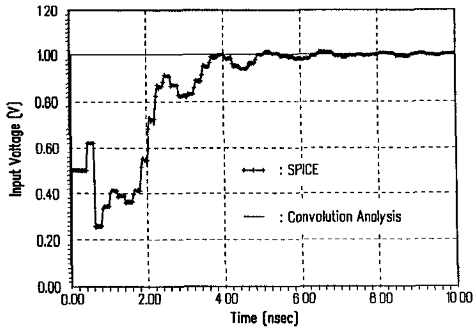
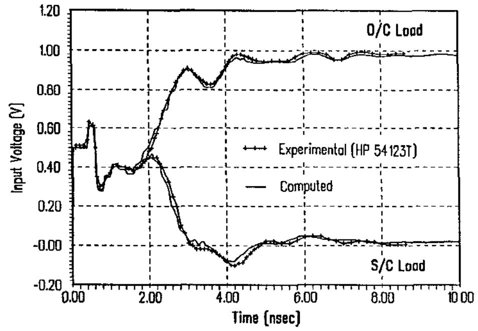


Fig. 4(b) Measured Step Response of Filter Compared to Convolution Analysis (with Loss)



Conclusions

A convolution-based method has been described for the transient analysis of causal linear systems. The main novelty is in the method for computing discrete impulse response samples, which give excellent frequency interpolation capabilities with respect to the original system function, even though comparatively few samples are used. Convolution operations are accordingly highly efficient, and results have been presented to validate the method both in comparison to theoretical analysis, an existing simulation program (SPICE), and experimental results from a lossy microstrip filter.

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